EXTRA PROBLEMS FOR HOMEWORK 11

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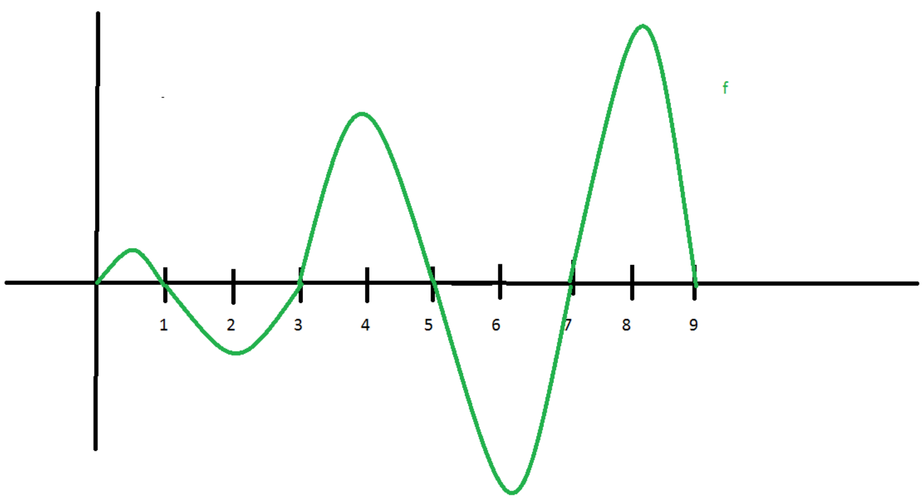
Problems

There are a total of 3 extra problems to do.

Problem 5.3.67. Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown.

(a) At what values of x do the local maximum and minimum values of g occur?
(b) Where does g attain its absolute maximum value?
(c) On what intervals is g concave downward?
(d) Sketch the graph of g

1A/Math 1A - Fall 2013/Homeworks/FTC.png



Note: My apologies for the badly drawn picture. I'm sure you can find a more accurate picture somewhere in your textbook (somewhere close to problem 67)

Date: Monday, December 2nd, 2013.

Problem 5.3.70. Calculate:

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$$

Problem 5.5.86: If f is continuous and $\int_0^9 f(x)dx = 4$, find $\int_0^3 xf\left(x^2\right)dx$

Solutions

Solution to 5.3.67.

(a) $g'(x) = f(x) = 0 \Rightarrow x = 1, 3, 5, 7, 9$, but 9 is an endpoints, so ignore it. Hence, by the second derivative test:

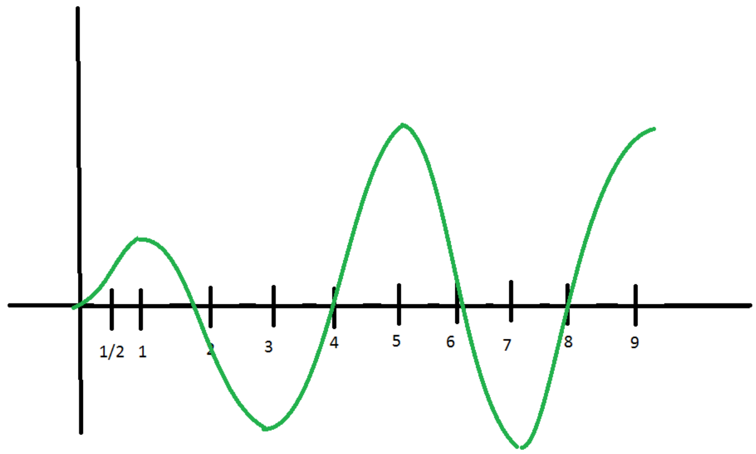
- $g''(1) = f'(1) < 0$, so g has a local max at 1
- $g''(3) = f'(3) > 0$, so g has a local min at 3
- $g''(5) = f'(5) < 0$, so g has a local max at 5
- $g''(7) = f'(7) > 0$, so g has a local min at 7

In summary, g attains a local minimum at $\boxed{3 \text{ and } 7}$, and a local maximum at $\boxed{1 \text{ and } 5}$.

(b) You do this by guessing. The candidates are $0, 1, 3, 5, 7, 9$ (critical points and endpoints). Notice $g(0) = 0, g(3) < 0$ but $g(5) > 0$, so you can eliminate 0 and 3. Also $g(5) > g(1)$, so you can eliminate 1. Also $g(7) < 0$, so you can eliminate 7. This leaves us with 5 and 9, but notice that $g(5) = g(9)$ (the areas between 5 and 9 cancel out), so the answer is $\boxed{x = 5 \text{ and } x = 9}$ (the book only writes $x = 9$, but I disagree)

(c) $g''(x) = f'(x)$, so to see where g is concave down, we have to check where $f'(x) < 0$, i.e. where f is decreasing. The answer is $\boxed{\left(\frac{1}{2}, 2\right) \cup (4, 6) \cup (8, 9)}$.

(d) 1A/Math 1A - Fall 2013/Homeworks/FTCSol.png



Solution to 5.3.70. First rewrite the limit as:

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\sqrt{\frac{i}{n}}$$

And you should recognize that $\Delta x = \frac{1}{n}$, $f(x) = \sqrt{x}$, $x_i = \frac{i}{n}$. In particular $a = x_0 = 0$ and $b = x_n = \frac{n}{n} = 1$, so in fact this limit equals to:

$$\int_0^1 \sqrt{x}dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1 = \frac{2}{3} - 0 = \frac{2}{3}$$

Solution to 5.5.86. Using the substitution $u = x^2$, we get $du = 2xdx$, so $xdx = \frac{1}{2}du$. Moreover, the endpoints become $u(0) = 0$ and $u(3) = 9$, so:

$$\int_0^3 xf\left(x^2\right)dx = \int_0^9 f(u)\frac{1}{2}du = \frac{1}{2}\int_0^9 f(x)dx = \frac{4}{2} = 2$$